Research on Data-driven Enterprise Raw Material Ordering and Optimal Transportation Scheme

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Abstract—Ordering and transportation optimization of raw material is a classic problem in the field of optimization. Based on the analysis, evaluation and prediction of previous data, this paper fully considers the actual situation of ordering, transshipment and storage, and establishes a multi-stage and large-scale mixed integer linear programming model for ordering and transporting raw materials. Combined with extensive data background, select appropriate indicators, and establish supplier evaluation systems by using entropy weight-CRITIC and RTOPSIS method. GM (1,1) is used to predict the general trend, the ARIMA model is used to predict the random fluctuation items, and a grey time series prediction model is constructed to obtain the predicted values of the data of the supply and loss rate in the next cycle. The prediction result are introduced into the planning model as parameters, and the evaluation score are used to construct a satisfaction function. The final goal of mixed integer linear programming model, as well as the three goals of sorting, transferring and storing, are obtained by reduction process. Finally, this paper uses Gurobi to solve practical problems, and obtains the ordering scheme and transshipment scheme that are superior to the historical schemes in ordering cost, transshipment loss and storage cost.

Index Terms—Supply and demand ordering decisions, Supplier evaluation, Grey time series prediction, Mixed-integer linear programming models, Gurobi

I. Introduction

Supply chain ordering decision, a hot issue in the field of optimization, has been widely studied by scholars at home and abroad. Van Weele[1] pointed out in 2005 that one of the key driver in the supply chain is procurement, and selecting the right supplier will affect the overall cost, which will determine the ordering decision to the greatest extent. Ravindran and Waring[2] pointed out in 2013 that inventory is the key driving factor that influences supply chain decision-making. Holding a large amount of inventory can make the supply chain respond to changes in demand faster, but it often leads to high costs. Therefore, inventory planning is an important issue in the ordering decision of supply chain.

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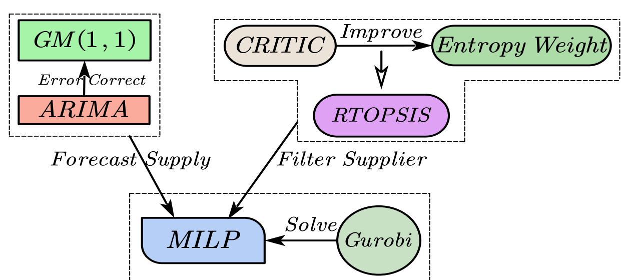
Heuristic algorithm may be an effective method for supply chain ordering decision-making, but Firouz Mohammad, Keskin B. and Melouk S.[3] found in 2017 that using heuristic algorithm based on decomposition to solve the problem of multi-supplier and single product would get poor decision-making. The latest work of Jos é A. Ventura et al.[4] (2022) provided two methods to solve the problem of multi-product and multi-cycle supplier selection and inventory lot size in serial supply chain—mixed integer linear programming method and sequence method, which have achieved good results, but they have not systematically studied the supplier's evaluation screening, historical prediction and optimization of the whole system.



Based on the consideration of ordering, transshipment and storage, this paper expands the previous work, establishes a multi-stage and large-scale mixed integer linear programming mathematical model for the decision-making of supplier-forwarder-company ordering scheme and transshipment scheme, and numerically solve the problem with actual data by Gurobi[5], which has been proved to be a global leading large-scale solution tool in the field of theory and practice.

II. Method

A. Evaluation of Suppliers

The evaluation of suppliers can be given priority, reducing the company's ordering range, simplify the ordering model and improve the efficiency of the solution.

1) Selection of Indicators: For the supply and demand relationship between enterprises and suppliers in historical data, four quantitative indicators are extracted and designed: supply stability, supply capacity, supply and demand fit, and supply and demand scale.

a) Stability Index:

$$\alpha_i = \frac{n_i}{N} \tag{1}$$

n_i represents the historical non-zero order number of each raw material supplier, and N represents the historical time span.

b) Supply Capacity Index:

$$\beta_i = \sqrt[3]{\frac{max_i}{Max}} \tag{2}$$

Max represents the global maximum availability among raw material suppliers, and max_i represents the maximum availability of each raw material supplier.

c) Supply and Demand Fitting Index: What the suppliers is most satisfied with is that the quantity of raw materials supplied is more in line with the order requirements, that is, the balance between supply and demand.

$$\gamma_i = |1 - mean|a_{ij}|| \tag{3}$$

$(a_{ij})_{n\times m}$ represents $(\frac{\text{Supply quantity-Order quantity}}{\text{Order quantity}})$ matrix, $mean||$ indicates mean value by row.

d) Supply Scale index:

$$\eta_i = \sqrt{\frac{avg_i}{max_i}} \tag{4}$$

avg_i represents the average value of the historical supply quantity of each supplier.

2) Determination of Weight: In order to give greater weight to indicators reflecting more information, the entropy-CRITIC method was used to assign weights to four indicators.

Calculate the entropy of each indicator:

$$e_j = -\frac{1}{\ln n} \sum_{i=1}^{n} \frac{b_{ij}}{\sum_{i=1}^{n} b_{ij}} \ln \frac{b_{ij}}{\sum_{i=1}^{n} b_{ij}} \tag{5}$$

By introducing its coefficient of variation $g_j = 1 - e_j$ into the CRITIC method, the calculation formula of the weights is obtained.

$$W_j = \frac{(\sigma_j + g_j) \times R_j}{\sum_{j=1}^{4} (\sigma_j + g_j) \times R_j} \tag{6}$$

where σ_j is the standard deviation, $R_j = \sum_{i=1}^{n} (1 - r_{ij})$, r_{ij} is the correlation coefficient.

3) Comprehensive Score: In order to eliminate the reverse order phenomenon that may arise from the general TOPSIS method, the RTOPSIS method is used to score each supplier.

Calculate the distance of each supplier and the absolutely ideal supplier and the negative ideal supplier:

$$\begin{aligned} D_i^* &= \sqrt{\sum_{j=1}^{n} w_j \left(C_{ij} - C_j^*\right)^2} \\ D_i^- &= \sqrt{\sum_{j=1}^{n} w_j \left(C_{ij} - C_j^-\right)^2} \end{aligned} \tag{7}$$

The comprehensive score was calculated as follows:

$$score_i = \frac{D_i^-}{D_i^* + D_i^-} \tag{8}$$

The $score_i$ reflects the importance of ith supplier to the business operations. In the subsequent process of supplier identification, the supplier with a high score will be given priority. In addition, the appropriate number of suppliers can be selected based on the score from high to low without considering all suppliers in the market, thus reducing the decision complexity and simplifying the ordering model.

B. Market Situation Forecast

For supply forecast, since most raw materials come from nature, the historical data of N time units are extracted according to the natural production cycle T of raw materials (determined according to the actual conditions), and the historical data are divided into $\frac{N}{T}$ groups. Each group of corresponding time data is extracted to form a non-negative original sequence.

$$Q_j^0 = \left\{ q_j^0(1), q_j^0(2), ..., q_j^0(\frac{N}{T}) \right\}, (j = 1, 2, \cdots, T)$$

Because time series prediction can better describe the random fluctuation characteristics of smooth series, it is not suitable for describing trend characteristics. Grey prediction can better describe the overall trend, but it is difficult to guarantee the accuracy when the data has the characteristics of random fluctuation. Therefore, GM (1,1) is used to predict the general trend, ARIMA model is used to predict the random fluctuation items, and a grey time series prediction model is constructed as follows:

Step 1: GM(1,1) is used to predict one bit of the sequence:

$$\hat{q}_j^1(k+1) = (q_j^1(1) - \frac{b}{a})e^{-a(k-1)} + \frac{b}{a} \tag{9}$$

A total $\frac{N}{T} + 1$ elements prediction sequence $\left\{ \hat{Q}_j^0 \right\}$ is obtained according to $\hat{q}_j^1(k+1) - \hat{q}_j^1(k)$; According to $e_j(t) = q_j^0(t) - \hat{q}_j^0(t)$, the residual sequence of $\frac{N}{T}$ elements is obtained.

Step 2: Differentiate the residual sequence until it is a smooth sequence. If it is a smooth series, then proceed to the next step. ADF test is used to determine whether the sequence is smooth. If the Test Statistic value is less than Critical View and the significance level below 0.05 is satisfied, the sequence is considered smooth.

Step 3: The ARIMA (p, q, d) model is used to predict the residual series. The value d is the number of differences in Step 2, and the autoregressive order p and the moving average order q of the smooth series are determined after d differences, so that the AIC index is as small as possible.

$$AIC = e^{\left(\frac{2k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T} \tag{10}$$

Therefore, the residual prediction sequence of $\{\hat{e}_j(t)\}$ with a total of $\frac{N}{T}+1$ elements is obtained.

Step 4: Add the corresponding elements of the prediction sequence $\left\{\hat{Q}_j^0\right\}$ and the residual prediction sequence $\{\hat{e}_j(t)\}$ to get the final prediction sequence $\left\{\tilde{Q}_j^0\right\}$. The final predicted value of one bit is:

$$\tilde{q}_j^0(\frac{N}{T}+1) = \hat{q}_j^0(\frac{N}{T}+1) + \hat{e}_j(\frac{N}{T}+1) \tag{11}$$

Step 5

Make the above prediction on T non-negative original sequences to obtain the prediction data of the next cycle:

$$y_j = \left\{\tilde{q}_j^0(\frac{N}{T}+1)\right\}, (j = 1, 2, \cdots, T) \tag{12}$$

For each supplier, the supply quantity: y_{ij} at each time in the next cycle can be predicted according to the above method. The prediction of the loss rate: λ_{ik} can be carried out in the same way as the supply.

C. Initial Planning Model

The following planning model for a future time period will be analyzed from three perspectives, divided into three parts: the ordering-supply situation of suppliers and companies, the forwarding situation of forwarders, and the storage and production-consumption situation of companies.

1) Ordering-Supply Situation:

$$Obj_1 = \min \sum_{i=1}^{T} (Pr_A \cdot \sum_{j\in\{A\}} x_{ij} + Pr_B \cdot \sum_{j\in\{B\}} x_{ij} + Pr_C \cdot \sum_{j\in\{C\}} x_{ij} + \cdots) \tag{13}$$

$$0 \le x_{ij} \le y_{ij} \tag{14}$$

The objective function (13) means to minimize the company's ordering cost, $Pr_A, Pr_B, Pr_C, \cdots$ respectively represent the unit purchase price of each raw material, the independent variable x_{ij} denotes the ordering quantity of raw materials from the jth supplier at the ith time, y_{ij} indicates the availability of the jth supplier at the ith time. The constraint (14) ensures that the order quantity of the jth supplier at time i does not exceed its supply capacity.

2) Transit Situation:

$$Obj_2 = \min \sum_{i=1}^{T} \sum_{j=1}^{S} \sum_{k=1}^{F} Z_{ijk} \lambda_{ik} \tag{15}$$

$$s.t. \begin{cases} \sum_{k=1}^{F} Z_{ijk} = y_{ij} \\ \sum_{j=1}^{S} Z_{ijk} \le F_{Max} \\ \sum_{k=1}^{F} \lceil \frac{Z_{ijk}}{M} \rceil = \lceil \frac{y_{ij}}{F_{Max}} \rceil \end{cases} \tag{16}$$

The objective function (15) is meant to minimize the loss in the developed transshipment scheme, the independent variable Z_{ijk} indicates the shipment quantity selected by the jth supplier at the ith time for the kth transshipment supplier, λ_{ik} means the loss rate of the kth transshipment supplier in the ith time, which can be obtained from the previous forecast, F_{Max} represents the weekly transport capacity limit of any freight forwarder at each time unit. To deliver all the supplier's goods, at least $\lceil \frac{y_{ij}}{F_{Max}} \rceil$ forwarders are needed, i.e., $\sum_{k=1}^{F} \lceil \frac{Z_{ijk}}{M} \rceil$ (M is a sufficiently large positive number).

3) Storage and Production Consumption:

$$Obj_3 = \min \sum_{t=1}^{T} Mo\Big[\Big(u_0^{(1)} + \cdots\Big) + \sum_{i=1}^{t} \Big(\sum_{j=1}^{S} \sum_{k=1}^{F} Z_{ijk}\left(1-\lambda_{ik}\right) - \Big(u_i^{(1)} + \cdots\Big)\Big)\Big] \tag{17}$$

The objective function (17) aims to minimize the total storage cost of S suppliers in T time. Assume that the storage amount of each raw material at the initial time is $u_0 = [u_0^{(1)}, u_0^{(2)}, \cdots]$, and the volume consumed at the ith time is $u_i = [u_i^{(1)}, u_i^{(2)}, \cdots]$, and Mo denotes the unit storage cost of raw materials.

At the same time, the company has a maximum production demand of D_{max} and a minimum production demand of D_{min} in each time unit, and each raw material it receives has a corresponding conversion rate of $Tr = [Tr_A, Tr_B, \cdots]$. In order to ensure the production demand of the company in each time unit, there are constraints(18):

$$D_{min} \le Tr \cdot u_i^T \le D_{max} \tag{18}$$

In practice, in order to maintain normal production, the company keeps the production demand of not less than N time as much as possible to reduce the risk, and N is adjusted according to different situations, taking the storage amount of each raw material as an element of vector U, namely:

$$U = \left[u_0^{(1)} + \sum_{t=1}^{i} \left(\sum_{j\in A} \sum_{k=1}^{F} Z_{tjk}\left(1-\lambda_{tk}\right) - u_t^{(1)}\right), \cdots\right]$$

So there is constraints(19):

$$Tr \cdot U^T \ge N \times D_{min} \tag{19}$$

In summary, in the practical case, the multi-objective optimization problem can be expressed as:

$$\min \quad \boldsymbol{Obj}(\boldsymbol{x}) = \{Obj_1, Obj_2, Obj_3\}$$

$$s.t. \quad h_i(\boldsymbol{x}) = 0, \quad g_r(\boldsymbol{x}) \geq 0, \quad \boldsymbol{x} \in D$$

.

Among them, Obj represents the vector of the objective function, $h_i(\boldsymbol{x})$ represents the constraint condition of the ith equality, $g_r(\boldsymbol{x})$ represents the constraint condition of the rth inequality, and D represents the set of values taken by the independent variables.

Because the selected objective functions Obj_1, Obj_2, and Obj_3 have different amplitudes, evaluation indexes and function trends, it is not easy to get the absolute optimal solution. In addition, in the actual order supply situation, after considering various additional costs to ensure that the basic production conditions are met, the company often chooses as few suppliers as possible. Secondly, the satisfaction function D is introduced to transform the problem into a single objective programming problem, and the Pareto optimal solution set of the problem is solved.

4) Normalization of Multi-objective Planning Problems: For the targets Obj_1, Obj_2 and Obj_3, give the corresponding satisfaction function d_i respectively(It is a number between [0,1]. The greater the satisfaction, the closer its value is to 1).Because all three targets expect to achieve the minimum value, the functional relationship between d_i and Obj_i is described as monotone increasing function form:

$$d_i = \frac{Obj_i - \min Obj_i}{\max Obj_i - \min Obj_i}$$

where $\min Obj_i$ and $\max Obj_i$ are determined by the historical data after corresponding processing.

In addition, note that the column vector of the composite evaluation index of the jth supplier is t_j and take:

$$d_4 = 1 - \frac{mean(x_j \cdot t_j)}{\sum t_j} \tag{20}$$

Where $x_j = \lceil \frac{x_{ij}}{M} \rceil$, M is a sufficiently large positive number.

Solving the above multi-objective programming problem results in Z_{ijk} and x_{ij}. After the transformation, the transfer scheme and sorting scheme are obtained respectively. In addition, in practice, for the convenience of statistics, the company often sets the ordering scheme and transshipment scheme as integers, so this model is a mixed integer linear programming model. Furthermore, in practice situations, in order to facilitate statistics, the company often sets the ordering scheme and transshipment scheme as integers, so this model is a mixed integer linear programming model.

III. Numerical Solution

In this paper, the actual data set of problem C of CUMCM[6] in 2021 is used to solve the numerical problem.Including the order data and self-supply data of 402 suppliers supplying raw materials A, B and C in the past 240 weeks, as well as the transshipment loss data of 8 forwarders. Some unknown parameters and some hypothetical parameters in the previous model given in the question are shown in table I.

TABLE I
The parameters to be determined in the problem

Symbol	Numerical Values	Symbol	Numerical Values
S	402 Pcs	T	24 Week
F	8 Pcs	F_{Max}	$6000m^3$
D_{max}	$28200m^3$	D_{min}	$24162m^3$
u_0	$48324m^3$	M_0	1 Unit Cost
Tr_A	$\frac{1}{0.6}$	Pr_A	1.2Unit Cost
Tr_B	$\frac{1}{0.66}$	Pr_B	1.1Unit Cost
Tr_C	$\frac{1}{0.72}$	Pr_C	1Unit Cost

A. Evaluation Result

Based on the model constructed in the previous section, for this problem, the historical ordering and supply data of 402 suppliers are processed accordingly to obtain the values of four indicators (stability, supply capacity, supply-demand fit, and supply scale) of each supplier, and the indicators are assigned on Python using the objective assignment method of entropy weight - CRITIC, the weighting results are shown in table II.

TABLE II
Index Weight

Stability	Capacity	Supply&Demand Fitting	Scale
0.3498	0.2269	0.1930	0.2302

From the above table, it can be seen that the weight of stability is the largest, the weight of supply capacity and supply scale is small and similar, and the weight of supply and demand fit index is the smallest.

Using the weighted data, the scores of 402 suppliers can be obtained based on the RTOPSIS model in the previous section so as to get the ranking. figure 1 shows the top 11 suppliers and their scores, in which the scores are converted into the range of 0-100.

B. Prediction Result

For the subsequent planning modeling, based on the previous GM (1,1) -ARIMA model, the supply situation of 402 suppliers for the next 24 weeks was predicted, and the loss rate of 8 forwarders for the next 24 weeks was also predicted based on this model.

Figure 2 shows the predicted supply of two suppliers S140 and S347 with high scores in the evaluation model

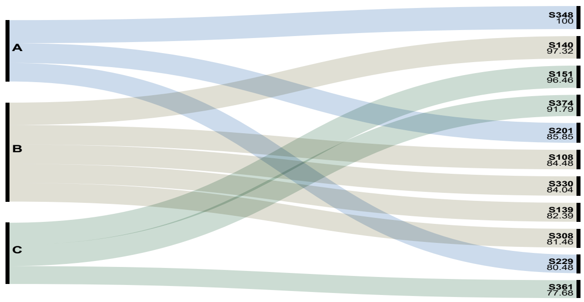


Fig. 1. Ranking and Score

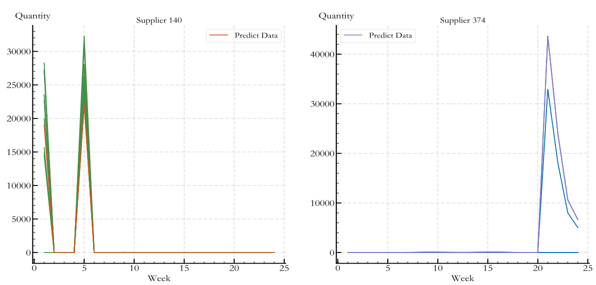


Fig. 2. Predicted Result

in the next 24 weeks, and the supply data for the past 240 weeks and 10 phases.

As can be seen from the figure, the GM (1,1) -ARMIA model has a good fit for the periodic and trend terms and a good fit for the abrupt variability, and in general, the model performs relatively well, so the prediction results can be applied to the next planning model.

C. Planning Result

Gurobi solver has been proved to be a global leading large-scale solution tool in theory and practice. The final planning model is numerically solved by Gurobi (version 9.5.0), so as to obtain ordering and transfer scheme of a small group of 402 suppliers and 50 suppliers in the next 24 weeks (small-scale problems allow adjustment and improvement of model and process).

Table III shows the results of partial ordering scheme of 402 suppliers.

TABLE III
Partial data of 402 suppliers' supply schemes

Weeks / Suppliers	Week1	⋯	Week7	Week8	⋯	Week24
S1	0	⋯	1	1	⋯	1
⋯			⋯	⋯		
S66	3	⋯	8	1	⋯	0
⋯			⋯	⋯		
S140	15933	⋯	0	11	⋯	0
⋯			⋯	⋯		
S229	0	⋯	1322	2467	⋯	0
⋯			⋯	⋯		
S348	0	⋯	0	785	⋯	85
⋯			⋯	⋯		

Table IV shows some data of 402 suppliers' transshipment plans (the quantity of raw material transferred by the sixth forwarders selected in 24 weeks).

TABLE IV
Selected transit data from 402 suppliers

Suppliers No.	Forwarder 6					
	Week1	Week2	Week3	⋯	Week23	Week24
S40	0	0	0	⋯	2	0
⋯				⋯		
S140	5735	0	0	⋯	0	0
⋯				⋯		
S151	0	0	1090	⋯	728	0
⋯				⋯		
S348	0	0	0	⋯	84	85
⋯				⋯		

IV. Results Discussion

In order to verify whether Gurobi has found an excellent scheme, historical data is taken as the standard. In practical problems, the smaller the first three objective functions, the better. Under the condition of table I, here $\min Obj_I$ is set to 0, $\max Obj_i$ is set to the minimum historical value in 240 weeks:

(1) Historical minimum ordering cost: for each 24-week cycle, the suppliers of the three raw materials ordered are processed separately, the supply quantities are added and multiplied by their corresponding ordering costs (unit costs), and then the ordering costs for 10 24-week cycles are added, and finally the minimum value is obtained.

(2) Minimum historical transportation loss: in every 24 weeks of history, find out the average weekly loss rate and the total supply, multiply each item, then add up to get 10 24-week transportation losses, and finally get the minimum value.

(3) Historical minimum storage cost: Firstly, from the table I, the inventory at the beginning of the next 24 weeks in the planning model is $u_0 = 48324m^3$,and it is worthwhile to assume that the initial inventory is of this value in each 24 weeks of history, and that the production consumes $24126m^3$per week (after raw material conversion), similar to the objective function (17) can be found for each 24-week storage cost, and finally take the minimal value to get.

The plan also provides the 50 suppliers with the highest scores in the evaluation model with ordering and forwarding options for the next 24 weeks. For these two solutions, the four target values are compared with the historical minimum values and the historical average values described earlier (the average value is taken only when extreme values is taken instead of the average value), as shown in figure 3, where the historical minimum values is set as the unit values.

It can be seen from the figure that, for the two planning results, the expected values of the order solution and transit solution for the next 24 weeks are all smaller than

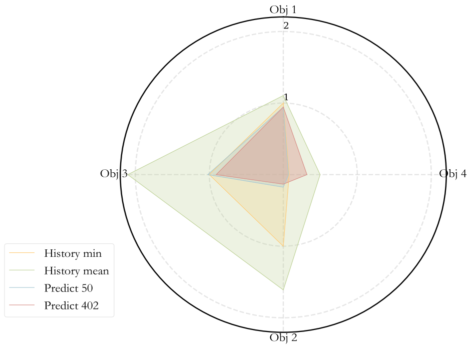


Fig. 3. Objection Comparison

the historical average data in terms of order cost, transit loss and storage cost to a large extent, and are both smaller than the historical minimum value in terms of order cost and transit loss. For the fourth goal, the "History mean" is the calculated value of the historical data of 402 suppliers according to the formula 20, while the "History min" is about the top 50 suppliers, and it can be found that the two solved goal values are less than the historical values. To sum up, it shows that Gurobi found the solution with excellent performance.

The work in this paper is compared with some previous work. Taking the work of Jos é A. Ventura et al[4]. Mentioned in the introduction as an example, they put forward the integration method and sequence method of mixed integer linear programming model, both of which achieved good results. According to the actual problems of 30 suppliers, the total cost was reduced by about 36.37% and 37.21% respectively, and the solving time was basically controlled at about 20 seconds. The solution information in this article is displayed in the tableV and figure3. Our work is targeted at 402 suppliers and 50 suppliers. On the premise of historical data, the total order cost, transfer cost and storage cost are reduced by at least 50%, while the settlement time of 50 suppliers is about 17 seconds. The solution effect is good and the solution speed is high.

TABLE V
Problem Scale and Solution Information

Statistical Information	50	402
NumVars	24024	192984
NumConstrs	63864	511608
Time	17.88s	42.35s
Iterations	104880	49383

Although there are differences in details between the specific work of Jos é A. Ventura et al[4], from the performance of the results, it can also be seen that the supplier decision system of screening-forecasting-optimization proposed in this paper has the characteristics of stable model and excellent solving ability.

V. Conclusion and Future Research

In this paper, under the background of big data, the future decision-making problem of raw material ordering scheme and transshipment scheme of supplier-forwarder-company is studied. After supplier evaluation and screening, a multi-stage and large-scale mixed integer linear programming model is established based on grey time series prediction model, some historical data are properly predicted, and the actual problem conditions are numerically solved by using Gurobi model. Finally, a series of analysis and discussion are made on the results.

Most of the existing researches assume that the future data distribution is known, and use simulation to consider the solution. However, this paper establishes a grey time series prediction model with good result performance, and innovatively relates historical data. In addition, the paper makes the model more relevant to the actual situation by establishing the entropy-CRITIC RTOPSIS model, which is a rather objective evaluation method, to measure the supplier level to reduce the model size. In addition, this paper considers the storage situation when establishing the mathematical model, so that the model is more reasonable.In addition, the author encountered obstacles in solving the mixed-integer linear programming model using Python conventional library, so this paper uses Gurobi combined with the actual problem to solve the model, which is another innovation of this paper.

In summary, for the multi-stage, large-scale mixed-integer linear programming problem of ordering and transporting raw materials, this paper has innovated in terms of historical big data processing as well as solution schemes, and obtained excellent performing result schemes.

However, the limitation of this paper is that it doesn't take timely response into account, and it may not have good portability in some supplier decision scenarios. Furthermore, the problems solved in this paper have certain support for interdisciplinary application, and may be expanded in the future by combining with deep learning methods.

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